

STOCHASTIC DIFFERENTIAL EQUATIONS HARVESTING MODELS: SIMULATION AND NUMERICAL SOLUTION

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ABSTRACT

Harvesting is a fundamental activity that significantly influences both human society, by supplying food, and the environment, by consuming resources. Maintaining an equilibrium is key for ecosystem integrity and the continued viability of harvesting practices. Therefore, harvesting policies should aim to achieve this balance by implementing sustainable quotas, regulated by effort.

Effort, in fisheries, refers to the operational plan, quantifiable by factors such as the number of vessels, workers, or hours at sea. This inherent structure imposes significant rigidity on organizations due to the substantial costs associated with fluctuating effort levels. Consequently, highly volatile effort policies, while useful for guiding profit comparisons, are impractical for direct application, as seen, for instance, in Brites (2017), Brites and Braumann (2019, 2023).

This work determines an optimal variable effort by numerically solving a non-linear Partial Differential Equation. The resulting optimal variable effort yields higher profits, which serve as a reference for assessing the feasibility of various strategies through comparative analysis of their outcomes.

Following Reis and Brites (2024, 2025), we modelled prices using a quadratic function and a Geometric Brownian Motion, and incorporated a discount factor that varies over time. For the quadratic price function, a penalized effort strategy was also implemented to determine and assess the resulting effort behaviour and associated profit, comparing these against reference values.

A strength parameter, ε , significantly influences profit by penalizing substantial effort changes. Higher ε reduces profit and mitigates sharp oscillations, yielding a more stable, albeit constantly adjusted, effort. This approach addresses social issues (arising from low or zero efforts), while still presenting logistical challenges stemming from frequent oscillations.

Keywords: SDE, PDE, HJB equation, numerical methods.

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